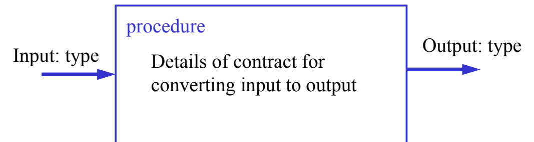
Today's topics

- Abstractions
  - Procedural
  - Data
- Relationship between data abstraction and procedures that operate on it
- Isolating use of data abstraction from details of implementation

Procedural abstraction

- **Process of procedural abstraction**
  - Define formal parameters, capture pattern of computation as a process in body of procedure
  - Give procedure a name
  - Hide implementation details from user, who just invokes name to apply procedure



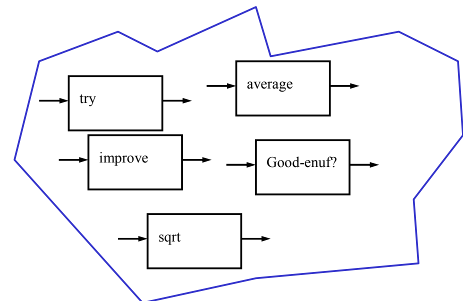
Procedural abstraction example: sqrt

To find an approximation of square root of x:

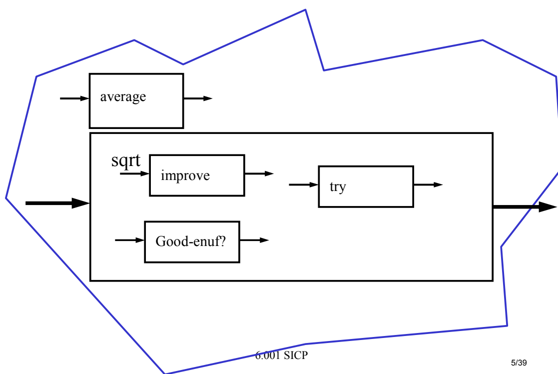
- Make a guess G
- Improve the guess by averaging G and x/G
- Keep improving the guess until it is good enough

```
(define try (lambda (guess x)
                  (if (good-enuf? guess x)
                      guess
                      (try (improve guess x) x))))
(define improve (lambda (guess x)
                      (average guess (/ x guess))))
(define average (lambda (a b) (/ (+ a b) 2)))
(define good-enuf? (lambda (guess x)
                       (< (abs (- (square guess) x)) 0.001)))
(define sqrt (lambda (x) (try 1 x)))
```


The universe of procedures for sqrt

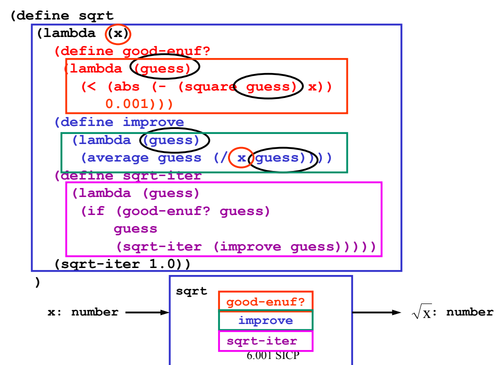


The universe of procedures for sqrt



sqrt - Block Structure

```
(define sqrt
 (lambda (x)
  (define good-enuf?
   (lambda (guess)
    (< (abs (- (square guess) x))
      0.001)))
  (define improve
   (lambda (guess)
    (average guess (/ x guess))))
  (define sqrt-iter
   (lambda (guess)
    (if (good-enuf? guess)
      guess
      (sqrt-iter (improve guess)))))
  (sqrt-iter 1.0))
)
```



Summary of part 1

- Procedural abstractions
  - Isolate details of process from its use
  - Designer has choice of which ideas to isolate, in order to support general patterns of computation

Language Elements

- Primitives
  - prim. data: numbers, strings, booleans
  - primitive procedures
- Means of Combination
  - procedure application
  - compound data (today)
- Means of Abstraction
  - naming
  - compound procedures
    - block structure
    - higher order procedures (next time)
  - conventional interfaces – lists (today)
  - data abstraction

Compound data

- Need a way of gluing data elements together into a unit that can be treated as a simple data element
- Need ways of getting the pieces back out

•Need a contract between the “glue” and the “unglue”

•Ideally want the result of this “gluing” to have the property of **closure:**

•“the result obtained by creating a compound data structure can itself be treated as a primitive object and thus be input to the creation of another compound object”

Pairs (cons cells)

- (cons <x-exp> <y-exp>) ==> <P>
  - Where <x-exp> evaluates to a value <x-val>, and <y-exp> evaluates to a value <y-val>
  - **Returns a** pair <P> whose car-part is <x-val> and whose cdr-part is <y-val>
- (car <P>) ==> <x-val>
  - Returns the car-part of the pair <P>
- (cdr <P>) ==> <y-val>
  - Returns the cdr-part of the pair <P>

Compound Data

- Treat a PAIR as a single unit:
  - Can pass a pair as argument
  - Can return a pair as a value

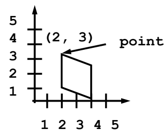
```
(define (make-point x y)
  (cons x y))

(define (point-x point)
  (car point))

(define (point-y point)
  (cdr point))

(define (make-seg pt1 pt2)
  (cons pt1 pt2))

(define (start-point seg)
   (car seg))
```



Pair Abstraction

- Constructor

```
; cons: A,B -> A X B
; cons: A,B -> Pair<A,B>
  (cons <x> <y>) ==> <P>
```

- Accessors

```
; car: Pair<A,B> -> A
  (car <P>) ==> <x>
; cdr: Pair<A,B> -> B
  (cdr <P>) ==> <y>
```

- Predicate

```
; pair? anytype -> boolean
  (pair? <z>)
   ==> #t  if <z> evaluates to a pair, else #f
```

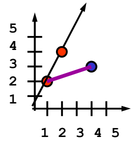
Pair abstraction

- Note how there exists a contract between the constructor and the selectors:
  - *(car (cons <a> <b>))* ➔ <a>
  - *(cdr (cons <a> <b>))* ➔ <b>
- Note how pairs have the property of closure – we can use the result of a pair as an element of a new pair:
  - *(cons (cons 1 2) 3)*

Using pair abstractions to build procedures

- Here are some data abstractions

```
(define p1 (make-point 1 2))
(define p2 (make-point 4 3))
(define s1 (make-seg p1 p2))
```




```
(define stretch-point
  (lambda (pt scale)
    (make-point
      (* scale (point-x pt))
      (* scale (point-y pt)))))
```

Constructor

Selector

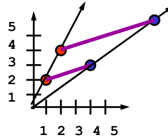
```
(stretch-point p1 2) ➔ (2 . 4)

p1 ➔ (1 . 2)
```


Using pair abstractions to build procedures

- Generalize to other structures

```
(define stretch-seg
 (lambda (seg sc)
  (make-seg (stretch-point (start-pt seg) sc)
            (stretch-point (end-pt seg) sc))))
```




```
(define seg-length
 (lambda (seg)
   (sqrt (+ (square (- (point-x (start-point seg))
                       (point-x (end-point seg))))
            (square (- (point-y (start-point seg))
                       (point-y (end-point seg))))))))
```

Selector for point

Selector for segment

Grouping together larger collections

- Suppose we want to group together a set of points. Here is one way

```
(cons (cons (cons (cons p1 p2)
                  (cons p3 p4))
            (cons (cons p5 p6)
                  (cons p7 p8)))
      p9)
```

- **UGH!!** How do we get out the parts to manipulate them?

Conventional interfaces -- lists

- A list is a data object that can hold an arbitrary number of ordered items.
- More formally, a list is a sequence of pairs with the following properties:
  - Car-part of a pair in sequence – holds an item
  - Cdr-part of a pair in sequence – holds a pointer to rest of list
  - Empty-list `nil` – signals no more pairs, or end of list
- Note that lists are closed under operations of `cons` and `cdr`.

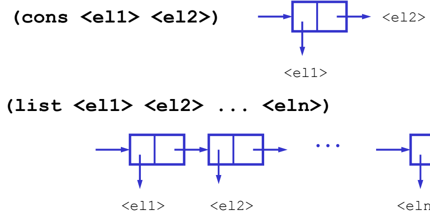
Conventional Interfaces - Lists

```
(cons <el1> <el2>)
```

<el2>

<el1>

```
(list <el1> <el2> ... <eln>)
```



Predicate

```
(null? <z>)
 ==> #t
```

if `<z>` evaluates to empty list

… to be really careful

- For today we are going to create different constructors and selectors for a list
  - `(define first car)`
  - `(define rest cdr)`
  - `(define adjoin cons)`
- Note how these abstractions inherit closure from the underlying abstractions!

Common patterns of data manipulation

- Have seen common patterns of procedures
- When applied to data structures, often see common patterns of procedures as well
  - Procedure pattern reflects recursive nature of data structure
  - Both procedure and data structure rely on
    - Closure of data structure
    - Induction to ensure correct kind of result returned

Common Pattern #1: cons'ing up a list

```
(define (enumerate-interval from to)
  (if (> from to)
      nil
      (adjoin from
              (enumerate-interval
                (+ 1 from)
                to))))
```

```
(e-i 2 4)
(if (> 2 4) nil (adjoin 2 (e-i (+ 1 2) 4)))
(if #f nil (adjoin 2 (e-i 3 4)))
(adjoin 2 (e-i 3 4))
(adjoin 2 (adjoin 3 (e-i 4 4)))
(adjoin 2 (adjoin 3 (adjoin 4 (e-i 5 4))))
(adjoin 2 (adjoin 3 (adjoin 4 nil)))
```

(adjoin 2 (adjoin 3))
4

(adjoin 2)
3 4

==> (2 3 4)
2 3 4

Common Pattern #2: cdr'ing down a list

```
(define (list-ref lst n)
  (if (= n 0)
      (first lst)
      (list-ref (rest lst)
                (- n 1))))
```

`(list-ref joe 1)`

Note how induction ensures that code is correct – relies on closure property of data structure

```
(define (length lst)
  (if (null? lst)
      0
      (+ 1 (length (rest lst)))))
```


Cdr'ing and Cons'ing Examples

```
(define (copy lst)
  (if (null? lst)
      nil                  ; base case
      (adjoin (first lst)  ; recursion
              (copy (rest lst)))))
```

```
(append (list 1 2) (list 3 4))
==> (1 2 3 4)
```

Strategy: "copy" list1 onto front of list2.

```
(define (append list1 list2)
  (cond ((null? list1) list2)   ; base
        (else
         (adjoin (first list1)     ; recursion
                 (append (rest list1)
                         list2)))))
```


Common Pattern #3: Transforming a List

```
(define group (list p1 p2 … p9))

(define stretch-group
  (lambda (gp sc)
    (if (null? gp)
        nil
        (adjoin (stretch-point (first gp) sc)
                (stretch-group (rest gp) sc)))))
```

Note how application of **`stretch-group`** to a list of points nicely separates the operations on points from the operations on the group, without worrying about details

Note also how this procedure walks down a list, creates a new point, and cons'es up a new list of points.

Common Pattern #3: Transforming a List

```
(define add-x (lambda (gp)
                (if (null? gp)
                    0
                    (+ (point-x (first gp))
                       (add-x (rest gp))))))
(define add-y (lambda (gp)
                (if (null? gp)
                    0
                    (+ (point-y (first gp))
                       (add-y (rest gp))))))
(define centroid (lambda (gp)
                   (let ((x-sum (add-x gp))
                         (y-sum (add-y gp))
                         (how-many (length gp)))
                     (make-point (/ x-sum how-many)
                                 (/ y-sum how-many)))))
```


Lessons learned

- There are conventional ways of grouping elements together into compound data structures.
- The procedures that manipulate these data structures tend to have a form that mimics the actual data structure.
- Compound data structures rely on an inductive format in much the same way recursive procedures do. We can often deduce properties of compound data structures in analogy to our analysis of recursive procedures by using induction.

Pairs, Lists, & Trees

Trees	• user *knows* trees are also lists
Lists	• user *knows* lists are also pairs
Pairs	• contract for `cons, car, cdr`

- Conventions that enable us to think about lists and trees.
- Specified the implementations for lists and trees: weak *data abstraction*
- How to build stronger abstractions?

Elements of a Data Abstraction

-- Pair Abstaction --

1. Constructor
```
;cons: A, B -> Pair<A,B>; A & B = anytype
(cons <x> <y>) ==> <p>
```
2. Accessors
```
(car <p>)          ; car: Pair<A,B> -> A
(cdr <p>)          ; cdr: Pair<A,B> -> B
```
3. Contract
```
(car (cons <x> <y>)) ==> <x>
(cdr (cons <x> <y>)) ==> <y>
```
4. Operations
```
; pair?: anytype -> boolean
(pair? <p>)
```
5. Abstraction Barrier

   Say nothing about implementation!

6. Concrete Representation & Implementation
```
Could have alternative implementations!
```


Rational numbers as an example

- A rational number is a ratio n/d
- a/b + c/d = (ad + bc)/bd
- a/b * c/d = (ac)/(bd)

Rational Number Abstraction

1. Constructor
```
; make-rat: integer, integer -> Rat
(make-rat <n> <d>) -> <r>
```
2. Accessors
```
; numer, denom: Rat -> integer
(numer <r>)
(denom <r>)
```
3. Contract
```
(numer (make-rat <n> <d>)) ==> <n>
(denom (make-rat <n> <d>)) ==> <d>
```
4. Layered Operations
```
(print-rat <r>) prints rat
(+rat x y) ; +rat: Rat, Rat -> Rat
(*rat x y) ; *rat: Rat, Rat -> Rat
```
5. Abstraction Barrier
```
Say nothing about implementation!
```

Rational Number Abstraction

1. Constructor
2. Accessors
3. Contract
4. Layered Operations
5. Abstraction Barrier

6. Concrete Representation & Implementation

```
; Rat = Pair<integer,integer>
(define (make-rat n d) (cons n d))
(define (numer r) (car r))
(define (denom r) (cdr r))
```


Alternative Rational Number Abstraction

1. Constructor
2. Accessors
3. Contract
4. Layered Operations
5. Abstraction Barrier

6. Concrete Representation & Implementation

```
; Rat = List
(define (make-rat n d) (list n d))
(define (numer r) (car r))
(define (denom r) (cadr r))
```


print-rat Layered Operation

```
; print-rat: Rat -> undef
(define (print-rat rat)
  (display (numer rat))
  (display "/")
  (display (denom rat)))
```


Layered Rational Number Operations

```
; +rat: Rat, Rat -> Rat
(define (+rat x y)
  (make-rat (+ (* (numer x) (denom y))
               (* (numer y) (denom x)))
            (* (denom x) (denom y))))

; *rat: Rat, Rat -> Rat
(define (*rat x y)
  (make-rat (* (numer x) (numer y))
            (* (denom x) (denom y))))
```


Using our system

- (define one-half (make-rat 1 2))
- (define three-fourths (make-rat 3 4))
- (define new (+rat one-half three-fourths))

```
(numer new) ➔ 10
(denom new) ➔ 8
```

Oops – should be 5/4 not 10/8!!

"Rationalizing" Implementation

```
(define (gcd a b)
  (if (= b 0)
    a
    (gcd b (remainder a b))))
```

Strategy: remove common factors when access numer and denom

```
(define (numer r)
  (let ((g (gcd (car r) (cdr r))))
    (/ (car r) g)))

(define (denom r)
  (let ((g (gcd (car r) (cdr r))))
    (/ (cdr r) g)))

(define (make-rat n d)
  (cons n d))
```

Alternative "Rationalizing" Implementation

- Strategy: remove common factors when create a rational number

```
(define (numer r) (car r))

(define (denom r) (cdr r))

(define (make-rat n d)
  (let ((g (gcd n d)))
    (cons (/ n g)
          (/ d g))))

(define (gcd a b)
  (if (= b 0)
    a
    (gcd b (remainder a b))))
```


Alternative +rat Operations

```
(define (+rat x y)
  (make-rat (+ (* (numer x) (denom y))
               (* (numer y) (denom x)))
            (* (denom x) (denom y))))

(define (+rat x y)
  (cons (+ (* (car x) (cdr y))
           (* (car y) (cdr x)))
        (* (cdr x) (cdr y))))

(define (+rat x y)
  (let ((n (+ (* (car x) (cdr y))
              (* (car y) (cdr x))))
        (d (* (cdr x) (cdr y))))
    (let ((g (gcd n d)))
      (cons (/ n g)
            (/ d g)))))
```

Lessons learned

- Valuable to build strong abstractions
  - Hide details behind names of accessors and constructors
  - Rely on closure of underlying implementation
- Enables user to change implementation without having to change procedures that use abstraction
- Data abstractions tend to have procedures whose structure mimics their inherent structure