6.001 SICP
Interpretation part 1

- Parts of an interpreter
- Arithmetic calculator
- Names
- Conditionals and if
- Store procedures in the environment
- Environment as explicit parameter
- Defining new procedures

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Why do we need an interpreter?

- Abstractions let us bury details and focus on use of modules to solve large systems
- Need to unwind abstractions at execution time to deduce meaning
- Have seen such a process – Environment Model
- Now want to describe that process as a procedure

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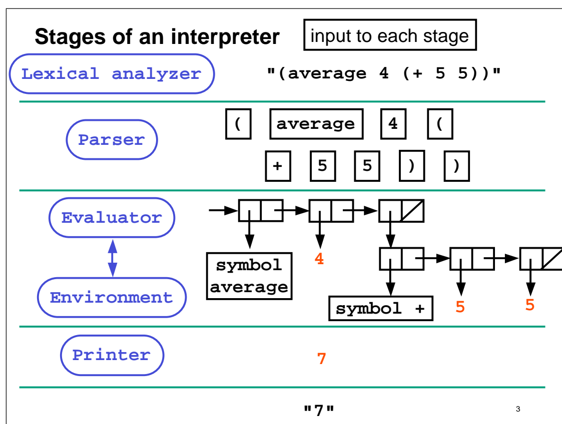
Stages of an interpreter



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Role of each part of the interpreter

- Lexical analyzer
  - break up input string into "words" called tokens
- Parser
  - convert linear sequence of tokens to a tree
  - like diagramming sentences in elementary school
  - also convert self-evaluating tokens to their internal values
    - **#f** is converted to the internal false value
- Evaluator
  - follow language rules to convert parse tree to a value
  - read and modify the environment as needed
- Printer
  - convert value to human-readable output string

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Goal of lecture

- Implement an interpreter for a programming language
- Only write evaluator and environment
  - use scheme's reader for lexical analysis and parsing
  - use scheme's printer for output
  - to do this, our language must look like scheme
- Call the language **scheme***
  - All names end with a star
- Start with a simple calculator for arithmetic
- Progressively add **scheme*** features
  - only get halfway there today

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1. Arithmetic calculator

Want to evaluate arithmetic expressions of two arguments, like:

```
(plus* 24 (plus* 5 6))
```

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1. Arithmetic calculator

```
(define (tag-check e sym) (and (pair? e) (eq? (car e) sym)))
(define (sum? e) (tag-check e 'plus*))

(define (eval exp)
  (cond
   ((number? exp) exp)
   ((sum? exp)     (eval-sum exp))
   (else
    (error "unknown expression " exp))))

(define (eval-sum exp)
   (+ (eval (cadr exp)) (eval (caddr exp))))


(eval '(plus* 24 (plus* 5 6)))
```

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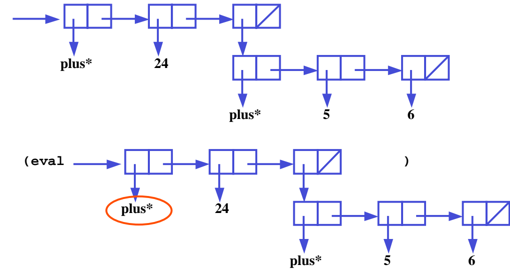
We are just walking through a tree …



sum? checks the tag

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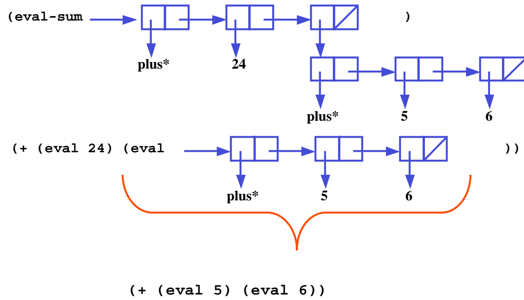
We are just walking through a tree …



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1. Arithmetic calculator

`(plus* 24 (plus* 5 6))`

- What are the argument and return values of **`eval`** each time it is called in the evaluation of line 17?

Call	Value	Call	Value
		(eval 5)	5
		(eval 6)	6
		(eval-sum '(plus* 5 6))	11
(eval 24)	24	(eval '(plus* 5 6))	11
(eval-sum '(plus* 24 (plus* 5 6)))	35		
(eval '(plus* 24 (plus* 5 6)))	35		

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1. Things to observe

- **`cond`** determines the expression type
- no work to do on numbers
  - scheme's reader has already done the work
  - it converts a sequence of characters like "24" to an internal binary representation of the number 24
- **`eval-sum`** recursively calls **`eval`** on both argument expressions

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2. Names

- Extend the calculator to store intermediate results as named values

  `(define* x* (plus* 4 5))` store result as x*

  `(plus* x* 2)` use that result

- Store bindings between names and values in a table
- What are the argument and return values of **`eval`** each time it is called in lines 34 and 35?
  - Show the environment each time it changes during evaluation of these two lines.

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2. Names

```
(define (define? exp) (tag-check exp 'define*))

(define (eval exp)
  (cond
   ((number? exp) exp)
   ((sum? exp)     (eval-sum exp))
   ((symbol? exp) (lookup exp))
   ((define? exp) (eval-define exp))
   (else
    (error "unknown expression " exp))))

; table ADT from prior lecture:
; make-table         void -> table
; table-get          table, symbol -> (binding | null)
; table-put!         table, symbol, anytype -> undef
; binding-value      binding -> anytype

(define environment (make-table))
```

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2. Names …

```
(define (lookup name)
  (let ((binding (table-get environment name)))
    (if (null? binding)
        (error "unbound variable: " name)
        (binding-value binding))))

(define (eval-define exp)
  (let ((name          (cadr exp))
        (defined-to-be (caddr exp)))
    (table-put! environment name (eval defined-to-be))
    'undefined))

(eval '(define* x* (plus* 4 5)))
(eval '(plus* x* 2))
```

How many times is **eval** called in these two evaluations?

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Evaluation of page 2 lines 34 and 35

```
(eval '(define* x* (plus* 4 5)))
    (eval '(plus* 4 5))
        (eval 4) ==> 4
        (eval 5) ==> 5
    ==> 9
==> undefined

(eval '(plus* x* 2))
    (eval 'x*) ==> 9
    (eval 2) ==> 2
==> 11
```

names	values
x*	9

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2. Things to observe

- Use scheme function **symbol?** to check for a name
  - the reader converts sequences of characters like **"x*"** to symbols in the parse tree
- Can use any implementation of the **table** ADT
- **eval-define** recursively calls **eval** on the second subtree but not on the first one
- **eval-define** returns a special undefined value

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3. Conditionals and if

- Extend the calculator to handle conditionals and if:
  (if* (greater* y* 6) (plus* y* 2) 15)

- **greater*** an operation that returns a boolean
- **if*** an operation that evaluates the first subexp, checks if value is true or false

- What are the argument and return values of **eval** each time it is called in line 32?

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3. Conditionals and If

```
(define (greater? exp) (tag-check exp 'greater*))
(define (if? exp)      (tag-check exp 'if*))

(define (eval exp)
  (cond ...
   ((greater? exp) (eval-greater exp))
   ((if? exp)      (eval-if exp))
   (else (error "unknown expression " exp))))

(define (eval-greater exp)
  (> (eval (cadr exp)) (eval (caddr exp))))

(define (eval-if exp)
  (let ((predicate   (cadr exp))
        (consequent  (caddr exp))
        (alternative (cadddr exp)))
    (let ((test (eval predicate)))
      (cond
       ((eq? test #t)  (eval consequent))
       ((eq? test #f)  (eval alternative))
       (else           (error "predicate not boolean: "
                              predicate))))))
(eval '(define* y* 9))
(eval '(if* (greater* y* 6) (plus* y* 2) 15))
```

Note: if* is stricter than Scheme's if

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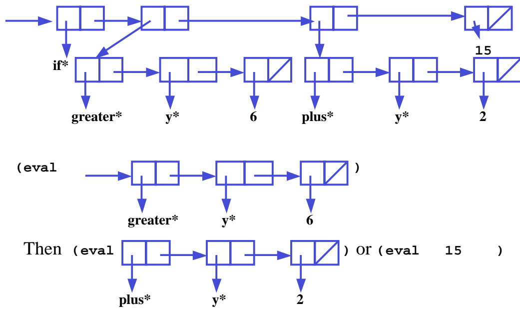
We are just walking through a tree …



Evaluation of page 3 line 32

```
(eval '(if* (greater* y* 6) (plus* y* 2) 15))
  (eval '(greater* y* 6))
     (eval 'y*) ==> 9
     (eval 6) ==> 6
  ==> #t
  (eval '(plus* y* 2))
     (eval 'y*) ==> 9
     (eval 2) ==> 2
  ==> 11
==> 11
```


3. Things to observe

- `eval-greater` is just like `eval-sum` from page 1
  - recursively call `eval` on both argument expressions
  - call scheme `>` to compute value
- `eval-if` does not call `eval` on all argument expressions:
  - call `eval` on the predicate
  - call `eval` on the consequent or on the alternative but not both

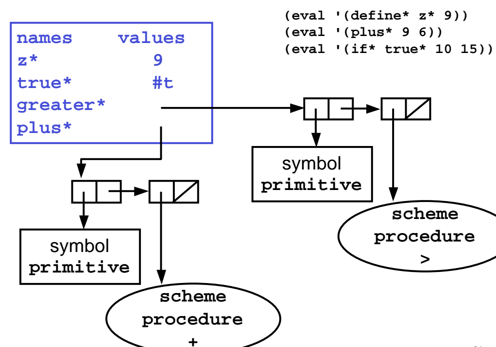
4. Store operators in the environment

- Want to add lots of operators but keep `eval` short
- Operations like `plus*` and `greater*` are similar
  - evaluate all the argument subexpressions
  - perform the operation on the resulting values
- Call this standard pattern an application
  - Implement a single case in `eval` for all applications
- Approach:
  - `eval` the first subexpression of an application
  - put a name in the environment for each operation
  - value of that name is a procedure
  - apply the procedure to the operands

4. Store operators in the environment

```
(define (application? e) (pair? e))

(define (eval exp)
  (cond
   ((number? exp)      exp)
   ((symbol? exp)      (lookup exp))
   ((define? exp)      (eval-define exp))
   ((if? exp)          (eval-if exp))
   ((application? exp) (apply (eval (car exp))
                              (map eval (cdr exp))))
   (else
    (error "unknown expression " exp))))

(define scheme-apply apply) ;; rename scheme's apply so we can reuse the name

(define (apply operator operands)
  (if (primitive? operator)
      (scheme-apply (get-scheme-procedure operator) operands)
      (error "operator not a procedure: " operator)))

;; primitive: an ADT that stores scheme procedures

(define prim-tag 'primitive)
(define (make-primitive scheme-proc)(list prim-tag scheme-proc))
(define (primitive? e)              (tag-check e prim-tag))
(define (get-scheme-procedure prim) (cadr prim))

(define environment (make-table))
(table-put! environment 'plus*    (make-primitive +))
(table-put! environment 'greater* (make-primitive >))
(table-put! environment 'true* #t)
```


Environment after eval 4 line 36



Evaluation of eval 4 line 37

```
(eval '(plus* 9 6))
(apply (eval 'plus*) (map eval '(9 6)))
(apply '(primitive #[add])
        (list (eval 9) (eval 6))
(apply '(primitive #[add]) '(9 6))
(scheme-apply
  (get-scheme-procedure '(primitive #[add]))
  '(9 6))
(scheme-apply #[add] '(9 6))
15
```


Evaluation of eval 4 line 38

```
(eval '(if* true* 10 15))
(eval-if '(if* true* 10 15))
(let ((test (eval 'true*))) (cond ...))
(let ((test (lookup 'true*))) (cond ...))
(let ((test #t)) (cond ...))
(eval 10)
10
```

Apply is never called!

4. Things to observe

- applications must be last case in `eval`
  - no tag check
- apply is never called in line 38
  - applications evaluate all subexpressions
  - expressions that need special handling, like `if*`, gets their own case in `eval`

5. Environment as explicit parameter

- change from
  `(eval '(plus* 6 4))`
  to
  `(eval '(plus* 6 4) environment)`
- all procedures that call `eval` have extra argument
- `lookup` and `define` use environment from argument
- No other change from evaluator 4
- Only nontrivial code: case for `application?` in eval

5. Environment as explicit parameter

```
(define (eval exp env)
  (cond
   ((number? exp)       exp)
   ((symbol? exp)      (lookup exp env))
   ((define? exp)      (eval-define exp env))
   ((if? exp)          (eval-if exp env))
   ((application? exp) (apply (eval (car exp) env)
                              (map (lambda (e) (eval e env))
                                   (cdr exp))))
   (else (error "unknown expression " exp))))

(define (lookup name env)
  (let ((binding (table-get env name)))
    (if (null? binding)
        (error "unbound variable: " name)
        (binding-value binding))))

(define (eval-define exp env)
  (let ((name (cadr exp))
        (defined-to-be (caddr exp)))
    (table-put! env name (eval defined-to-be env))
    'undefined))

(define (eval-if exp env)
  (let ((predicate   (cadr exp))
        (consequent  (caddr exp))
        (alternative (cadddr exp)))
    (let ((test (eval predicate env)))
      (cond
       ((eq? test #t)  (eval consequent env))
       ((eq? test #f)  (eval alternative env))
       (else           (error "predicate not boolean: "
                              predicate))))))
```

This change is boring! Exactly the same functionality as #4.

```
(eval '(define* z* (plus* 4 5))
      environment)
(eval '(if* (greater* z* 6) 10 15)
      environment)
```


6. Defining new procedures

- Want to add new procedures
- For example, a `scheme*` program:

```
(define* twice* (lambda* (x*) (plus* x* x*)))
(twice* 4)
```

- Strategy:
  - Add a case for `lambda*` to `eval`
    - the value of `lambda*` is a compound procedure
  - Extend `apply` to handle compound procedures
  - Implement environment model

6. Defining new procedures

```
(define (lambda? e) (tag-check e 'lambda*))

(define (eval exp env)
  (cond  ((number? exp)        exp)
         ((symbol? exp)       (lookup exp env))
         ((define? exp)       (eval-define exp env))
         ((if? exp)           (eval-if exp env))
         ((lambda? exp)       (eval-lambda exp env))
         ((application? exp) (apply (eval (car exp) env)
                                    (map (lambda (e) (eval e env))
                                         (cdr exp))))
         (else (error "unknown expression " exp))))

(define (eval-lambda exp env)
  (let ((args (cadr exp))
        (body (caddr exp)))
    (make-compound args body env)))

(define (apply operator operands)
  (cond ((primitive? operator)
         (scheme-apply (get-scheme-procedure operator) operands))
        ((compound? operator)
         (eval (body operator)
               (extend-env-with-new-frame
                           (parameters operator)
                           operands
                           (env operator))))
        (else (error "operator not a procedure: " operator))))

;; ADT that implements the "double bubble"
(define compound-tag 'compound)
(define (make-compound parameters body env)
                    (list compound-tag parameters body env))
(define (compound? exp)  (tag-check exp compound-tag))
(define (parameters compound) (cadr compound))
(define (body compound)       (caddr compound))
(define (env compound)        (cadddr compound))
```


Implementation of `lambda*`

```
(eval '(lambda* (x*) (plus* x* x*)) GE)
(eval-lambda '(lambda* (x*) (plus* x* x*)) GE)
(make-compound '(x*) '(plus* x* x*) GE)
(list 'compound '(x*) '(plus* x* x*) GE)
```

GE
symbol compound
symbol plus*
symbol x*

This data structure is a procedure!

Defining a named procedure

```
(eval '(define* twice*
         (lambda* (x*) (plus* x* x*))) GE)
```

names	values
z*	9
true*	#t
plus*	
twice*	

symbol primitive
scheme procedure +
symbol compound
symbol plus*
symbol x*

Implementation of `apply` (1)

```
(eval '(twice* 4) GE) some-other-environment)
(apply (eval 'twice* GE)
       (map (lambda (e) (eval e GE)) '(4)))
(apply (list 'compound '(x*) '(plus* x* x*) GE)
       '(4))
(eval '(plus* x* x*)
      (extend-env-with-new-frame '(x*) '(4) GE))
(eval '(plus* x* x*) E1)
```

A
E1
name	value
x*	4

GE

Implementation of `apply` (2)

```
(eval '(plus* x* x*) E1)
(apply (eval 'plus* E1)
       (map (lambda (e) (eval e E1)) '(x* x*)))
(apply '(primitive #[add]) (list (eval 'x* E1)
                                 (eval 'x* E1)))
(apply '(primitive #[add]) '(4 4))
(scheme-apply #[add] '(4 4))
8
```

After 8 is returned by `(eval '(plus* x* x*) E1)`, where is frame A stored?

A
E1
name	value
x*	4

GE

Implementation of environment model

- Environment = list<table>

A
E1
name	value
x*	4

GE

E1
GE

name	value
x*	4

name	value
plus*	(primitive #[add])
greater*	(primitive #[grt])
...	

; Environment model code (part of eval 6)

```
; Environment = list<table>

(define (extend-env-with-new-frame names values env)
  (let ((new-frame (make-table)))
    (make-bindings! names values new-frame)
    (cons new-frame env)))

(define (make-bindings! names values table)
  (for-each
     (lambda (name value) (table-put! table name value))
     names values))

; the initial global environment
(define GE
  (extend-env-with-new-frame
     (list 'plus* 'greater*)
     (list (make-primitive +) (make-primitive >))
     nil))

; lookup searches the list of frames for the first match
(define (lookup name env)
  (if (null? env)
      (error "unbound variable: " name)
      (let ((binding (table-get (car env) name)))
        (if (null? binding)
            (lookup name (cdr env))
            (binding-value binding)))))

; define changes the first frame in the environment
(define (eval-define exp env)
  (let ((name          (cadr exp))
        (defined-to-be (caddr exp)))
    (table-put! (car env) name (eval defined-to-be env))
    'undefined))

(eval '(define* twice* (lambda* (x*) (plus* x* x*))) GE)
(eval '(twice* 4) GE)
```

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Summary

- Cycle between eval and apply is the core of the evaluator
  - eval calls apply with operator and argument values
  - apply calls eval with expression and environment
  - no pending operations on either call
    - an iterative algorithm if the expression is iterative
- What is still missing from `scheme*` ?
  - ability to evaluate a sequence of expressions
  - data types other than numbers and booleans

Everything in these lectures would still work if you deleted the stars from the names!

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1. Arithmetic calculator

```
(define (tag-check e sym) (and (pair? e) (eq? (car e) sym)))
(define (sum? e) (tag-check e 'plus*))

(define (eval exp)
  (cond
   ((number? exp) exp)
   ((sum? exp)    (eval-sum exp))
   (else
    (error "unknown expression " exp))))

(define (eval-sum exp)
   (+ (eval (cadr exp)) (eval (caddr exp))))


(eval '(plus* 24 (plus* 5 6)))
```

2. Names

```
(define (define? exp) (tag-check exp 'define*))

(define (eval exp)
  (cond
   ((number? exp) exp)
   ((sum? exp)    (eval-sum exp))
   ((symbol? exp) (lookup exp))
   ((define? exp) (eval-define exp))
   (else
    (error "unknown expression " exp))))

; variation on table ADT from March 2 lecture (only difference is
; that table-get returns a binding, while original version
; returned a value):
; make-table        void -> table
; table-get         table, symbol -> (binding | null)
; table-put!        table, symbol, anytype -> undef
; binding-value     binding -> anytype

(define environment (make-table))

(define (lookup name)
  (let ((binding (table-get environment name)))
    (if (null? binding)
        (error "unbound variable: " name)
        (binding-value binding))))

(define (eval-define exp)
  (let ((name          (cadr exp))
        (defined-to-be (caddr exp)))
    (table-put! environment name (eval defined-to-be))
    'undefined))

(eval '(define* x* (plus* 4 5)))
(eval '(plus* x* 2))



; Index to procedures that have not changed:
;   procedure          page       line
;     sum?             1          4
;     eval-sum         1          13
```

3. Conditionals and if

```
(define (greater? exp) (tag-check exp 'greater*))
(define (if? exp)      (tag-check exp 'if*))

(define (eval exp)
  (cond
   ((number? exp)  exp)
   ((sum? exp)     (eval-sum exp))
   ((symbol? exp)  (lookup exp))
   ((define? exp)  (eval-define exp))
   ((greater? exp) (eval-greater exp))
   ((if? exp)      (eval-if exp))
   (else
    (error "unknown expression " exp))))

(define (eval-greater exp)
  (> (eval (cadr exp)) (eval (caddr exp))))

(define (eval-if exp)
  (let ((predicate   (cadr exp))
        (consequent  (caddr exp))
        (alternative (cadddr exp)))
    (let ((test (eval predicate)))
      (cond
       ((eq? test #t)  (eval consequent))
       ((eq? test #f)  (eval alternative))
       (else           (error "predicate not boolean: "
                              predicate))))))

(eval '(define* y* 9))
(eval '(if* (greater* y* 6) (plus* y* 2) 15))


; Index to procedures that have not changed:
;   procedure         page       line
;     sum?            1          4
;     eval-sum        1          13
;     lookup          2          22
;     define?         2          3
;     eval-define     2          28
```

4. Store operators in the environment

```
(define (application? e) (pair? e))

(define (eval exp)
  (cond
   ((number? exp)      exp)
   ((symbol? exp)      (lookup exp))
   ((define? exp)      (eval-define exp))
   ((if? exp)          (eval-if exp))
   ((application? exp) (apply (eval (car exp))
                              (map eval (cdr exp))))
   (else
    (error "unknown expression " exp))))

;; rename scheme's apply so we can reuse the name
(define scheme-apply apply)

(define (apply operator operands)
  (if (primitive? operator)
      (scheme-apply (get-scheme-procedure operator) operands)
      (error "operator not a procedure: " operator)))

;; primitive: an ADT that stores scheme procedures

(define prim-tag 'primitive)
(define (make-primitive scheme-proc)(list prim-tag scheme-proc))
(define (primitive? e)              (tag-check e prim-tag))
(define (get-scheme-procedure prim) (cadr prim))

(define environment (make-table))
(table-put! environment 'plus*    (make-primitive +))
(table-put! environment 'greater* (make-primitive >))
(table-put! environment 'true* #t)

(eval '(define* z* 9))
(eval '(plus* 9 6))
(eval '(if* true* 10 15))


; Index to procedures that have not changed:
;   procedure         evaluator  line
;     lookup          2          22
;     define?         2          3
;     eval-define     2          28
;     if?             3          4
;     eval-if         3          20
```

5. Environment as explicit parameter

;This change is boring! Exactly the same functionality as #4.

```
(define (eval exp env)
  (cond
   ((number? exp)       exp)
   ((symbol? exp)      (lookup exp env))
   ((define? exp)      (eval-define exp env))
   ((if? exp)          (eval-if exp env))
   ((application? exp) (apply (eval (car exp) env)
                              (map (lambda (e) (eval e env))
                                   (cdr exp))))
   (else
    (error "unknown expression " exp))))

(define (lookup name env)
  (let ((binding (table-get env name)))
    (if (null? binding)
        (error "unbound variable: " name)
        (binding-value binding))))

(define (eval-define exp env)
  (let ((name (cadr exp))
        (defined-to-be (caddr exp)))
    (table-put! env name (eval defined-to-be env))
    'undefined))

(define (eval-if exp env)
  (let ((predicate   (cadr exp))
        (consequent  (caddr exp))
        (alternative (cadddr exp)))
    (let ((test (eval predicate env)))
      (cond
       ((eq? test #t)  (eval consequent env))
       ((eq? test #f)  (eval alternative env))
       (else           (error "predicate not boolean: "
                              predicate))))))

(eval '(define* z* (plus* 4 5)) environment)
(eval '(if* (greater* z* 6) 10 15) environment)
```

Index to procedures that have not changed:

procedure	evaluator	line
define?	2	3
if?	3	4
application?	4	3
apply	4	19

6. Defining new procedures

```
(define (lambda? e) (tag-check e 'lambda*))

(define (eval exp env)
  (cond
   ((number? exp)       exp)
   ((symbol? exp)      (lookup exp env))
   ((define? exp)      (eval-define exp env))
   ((if? exp)          (eval-if exp env))
   ((lambda? exp)      (eval-lambda exp env))
   ((application? exp) (apply (eval (car exp) env)
                              (map (lambda (e) (eval e env))
                                   (cdr exp))))
   (else
    (error "unknown expression " exp))))

(define (eval-lambda exp env)
  (let ((args (cadr exp))
        (body (caddr exp)))
    (make-compound args body env)))

(define (apply operator operands)
  (cond ((primitive? operator)
         (scheme-apply (get-scheme-procedure operator)
                       operands))
        ((compound? operator)
         (eval (body operator)
               (extend-env-with-new-frame
                           (parameters operator)
                           operands
                           (env operator))))
        (else
         (error "operator not a procedure: " operator))))



;; ADT that implements the "double bubble"

(define compound-tag 'compound)
(define (make-compound parameters body env)
                    (list compound-tag parameters body env))
(define (compound? exp)  (tag-check exp compound-tag))

(define (parameters compound) (cadr compound))
(define (body compound)       (caddr compound))
(define (env compound)        (cadddr compound))
```

; Environment model code (part of eval 6)

```
; Environment = list<table>

(define (extend-env-with-new-frame names values env)
  (let ((new-frame (make-table)))
    (make-bindings! names values new-frame)
    (cons new-frame env)))

(define (make-bindings! names values table)
  (for-each
     (lambda (name value) (table-put! table name value))
     names values))

; the initial global environment
(define GE
  (extend-env-with-new-frame
     (list 'plus* 'greater*)
     (list (make-primitive +) (make-primitive >))
     nil))


; lookup searches the list of frames for the first match
(define (lookup name env)
  (if (null? env)
      (error "unbound variable: " name)
      (let ((binding (table-get (car env) name)))
        (if (null? binding)
            (lookup name (cdr env))
            (binding-value binding)))))

; define changes the first frame in the environment
(define (eval-define exp env)
  (let ((name          (cadr exp))
        (defined-to-be (caddr exp)))
    (table-put! (car env) name (eval defined-to-be env))
    'undefined))


(eval '(define* twice* (lambda* (x*) (plus* x* x*))) GE)
(eval '(twice* 4) GE)

Index to procedures that have not changed:
   procedure          evaluator  line
      define?         2          3
      if?             3          4
      application?    4          3
      eval-i
```