Today’s topic: Abstraction

• Procedural Abstractions

• Data Abstractions:
  • Isolate use of data abstraction from details of implementation

• Relationship between data abstraction and procedures that operate on it

Procedural abstraction

• Process of procedural abstraction
  • Define formal parameters, capture pattern of computation as a process in body of procedure
  • Give procedure a name
  • Hide implementation details from user, who just invokes name to apply procedure

Procedural abstraction example: sqrt

To find an approximation of square root of x:

• Make a guess G
• Improve the guess by averaging G and x/G
• Keep improving the guess until it is good enough

(define try (lambda (guess x)
  (if (good-enuf? guess x)
      guess
      (try (improve guess x) x))))

(define good-enuf? (lambda (guess x)
  (< (abs (- (square guess) x)) 0.001)))

(define improve (lambda (guess x)
  (average guess (/ x guess))))

(define average (lambda (a b) (/ (+ a b) 2)))

(define sqrt (lambda (x) (try 1 x)))

The universe of procedures for sqrt

sqrt - Block Structure

(define sqrt
  (lambda (x)
    (define good-enuf?
      (lambda (guess)
        (< (abs (- (square guess) x)) 0.001)))
    (define improve
      (lambda (guess)
        (average guess (/ x guess))))
    (define try
      (lambda (guess)
        (if (good-enuf? guess)
            guess
            (try (improve guess)))
      )
    )
    (try 1)
  )

sqrt: number -> number

Summary of part 1

• Procedural abstractions
  • Isolate details of process from its use
  • Designer has choice of which ideas to isolate, in order to support general patterns of computation
Language Elements

- Primitives
  - prim. data: numbers, strings, booleans
  - primitive procedures
- Means of Combination
  - procedure application
  - compound data (today)
- Means of Abstraction
  - naming
  - compound procedures
    - block structure
    - higher order procedures
  - conventional interfaces – lists
  - data abstraction

Compound data

- Need a way of (procedure for) gluing data elements together into a unit that can be treated as a simple data element
- Need ways of (procedures for) getting the pieces back out
- Need a contract between the "glue" and the "unglue"

- Ideally want the result of this "gluing" to have the property of closure:
  "the result obtained by creating a compound data structure can itself be treated as a primitive object and thus be input to the creation of another compound object"

Pairs (cons cells)

- \((\text{cons} \ <x\text{-exp}> \ <y\text{-exp}>) ==> <P>\)
  - Where \(<x\text{-exp}>\) evaluates to a value \(<x\text{-val}>\), and \(<y\text{-exp}>\) evaluates to a value \(<y\text{-val}>\)
  - Returns a pair \(<P>\) whose car-part is \(<x\text{-val}>\) and whose cdr-part is \(<y\text{-val}>\)
- \((\text{car} \ <P>) ==> <x\text{-val}>\)
  - Returns the car-part of the pair \(<P>\)
- \((\text{cdr} \ <P>) ==> <y\text{-val}>\)
  - Returns the cdr-part of the pair \(<P>\)

Pairs: Nano Quiz

- \((\text{define} \ pl \ (\text{cons} \ (+ \ 3 \ 2) \ 4))\)
  - \((\text{car} \ pl) ==> ?\)
  - \((\text{cdr} \ pl) ==> ?\)

Pairs Are A Data Abstraction

- Constructor
  - \(\text{cons}: A, B \rightarrow A \times B\)
  - \(\text{cons}: A, B \rightarrow \text{Pair} \langle A, B \rangle\)
  - \((\text{cons} \ <a> \ <b>) \rightarrow \langle P \rangle\)
- Accessors
  - \(\text{car}: \text{Pair} \langle A, B \rangle \rightarrow A\)
    - \((\text{car} \ <P>) ==> <a>\)
  - \(\text{cdr}: \text{Pair} \langle A, B \rangle \rightarrow B\)
    - \((\text{cdr} \ <P>) ==> <b>\)
- Contract
  - \(\langle \text{car} \ (\text{cons} \ <a> \ <b>) \rangle \rightarrow \star \ <a>\)
  - \(\langle \text{cdr} \ (\text{cons} \ <a> \ <b>) \rangle \rightarrow \star \ <b>\)
- Operations
  - \(\text{pair?}: \text{anytype} \rightarrow \text{boolean}\)
    - \((\text{pair?} \ <a>) \rightarrow \#t \ \text{if } \langle a \rangle \text{ evaluates to a pair, else } \#f\)

Pair Abstraction

- Once we build a pair, we can treat it as if it were a primitive (e.g. the same way we treat a number)
- Pairs have the property of closure, meaning we can use a pair anywhere we would expect to use a primitive data element:
  - \((\text{cons} \ (\text{cons} \ 1 \ 2) \ 3)\)
Building Additional Data Abstractions

```
(define (make-point x y)
  (cons x y))

(define (point-x point)
  (car point))

(define (point-y point)
  (cdr point))

(define P1 (make-point 2 3)
  (define P2 (make-point 4 1))

(define (make-seg pt1 pt2)
  (cons pt1 pt2))

(define (start-point seg)
  (car seg))

(define S1 (make-seg P1 P2))
```

Using Data Abstractions

```
(define pl (make-point 1 2))
(define p2 (make-point 4 3))
(define sl (make-seg pl p2))

(define stretch-point
  (lambda (pt scale)
    (make-point
      (* scale (point-x pt))
      (* scale (point-y pt))))))

(stretch-point pl 2) ⇒ (2 . 4)
pl ⇒ (1 . 2)
```

Grouping together larger collections

```
• Suppose we want to group together a set of points. Here is one way
  (cons (cons (cons (cons p1 p2)
                  (cons p3 p4))
         (cons (cons p5 p6)
              (cons p7 p8)))
  p9)

• UGH!! How do we get out the parts to manipulate them?
```

Conventional interfaces -- Lists

```
• A list is a data object that can hold an arbitrary number of ordered items.
• More formally, a list is a sequence of pairs with the following properties:
  • Car-part of a pair in sequence – holds an item
  • Cdr-part of a pair in sequence – holds a pointer to rest of list
  • Terminates in an empty-list `()` – signals no more pairs, or end of list
• Note that lists are closed under operations of `cons` and `cdr`.
```

Conventional Interfaces -- Lists

```
(cons <el1> <el2>)

(list <el1> <el2> ... <eln>)

(list 1 2 3 4) ⇒ (1 2 3 4)
```

Predicate

```
(null? <z>)
⇒ #t if <z> evaluates to empty list
```
... to be really careful

- For today we are going to create different constructors and selectors for a list, to distinguish from pairs ...
  - `(define first car)`
  - `(define rest cdr)`
  - `(define adjoin cons)`
- These abstractions inherit closure from the underlying abstractions

Common patterns of data manipulation

- Have seen common patterns of procedures
- When applied to data structures, often see common patterns of procedures as well
  - Procedure pattern reflects recursive nature of data structure
  - Both procedure and data structure rely on
    - Closure of data structure
    - Induction to ensure correct kind of result returned

Common pattern #1: cons'ing up a list

- Motivation:
  ```scheme
  (define 1thru4 (lambda() (list 1 2 3 4)))
  (define (2thru7) (list 2 3 4 5 6 7))
  ...
  ```

```scheme
(define (enumerate-interval from to)
  (if (> from to)
     '()
     (adjoin from (enumerate-interval (+ 1 from) to)))))
```

```scheme
(e-i 2 4)
(if (> 2 4) '() (adjoin 2 (e-i (+ 1 2) 4)))
(adjoin 2 (adjoin 3 (e-i 4 4)))
(adjoin 2 (adjoin 3 (adjoin 4 '())))
=> (2 3 4)
```

Common pattern #2: cdr'ing down a list

```scheme
(define (list-ref lst n)
  (if (= n 0)
     (first lst)
     (list-ref (rest lst) (- n 1)))))
```

```scheme
(joe)
(joe 1)
Note how induction ensures that code is correct – relies on closure property of data structure
```

```scheme
(define (length lst)
  (if (null? lst)
     0
     (+ 1 (length (rest lst)))))
```

Cdr'ing and Cons'ing Examples

```scheme
(define (copy lst) (if (null? lst) ; test
    '()
    (adjoin (first lst) ; base case
      (copy (rest lst)))))
```

```scheme
(append (list 1 2) (list 3 4))
=> (1 2 3 4)
```

Strategy: “copy” list1 onto front of list2.

```scheme
(define (append list1 list2) (cond ((null? list1) list2) ; base
    (else
      (adjoin (first list1) ; recursion
        (append (rest list1) list2)))))
```
Some facts of lists

1. Lists are (mostly) one-way data structures
   (define x (list 2 3 4))
   (car x) => ?
   (car (cdr x)) => ?

2. (cdr (cdr (cdr (cdr x)))) => ?

Common Pattern #3: Transforming a List

(define group (list p1 p2 ... p9))
(define stretch-group
  (lambda (gp sc)
    (if (null? gp)
      ()
      (adjoin (stretch-point (first gp) sc)
        (stretch-group (rest gp) sc)))))

stretch-group separates operations on points from operations on the group
Walks (cdr’s) down the list, creates a new point, cons’es up a new list of points.

Lessons learned

• There are conventional ways of grouping elements together into compound data structures.
• The procedures that manipulate these data structures tend to have a form that mimics the actual data structure.
• Compound data structures rely on an inductive format in much the same way recursive procedures do. We can often deduce properties of compound data structures in analogy to our analysis of recursive procedures by using induction.

Elements of a Data Abstraction

1. Constructors
2. Accessors
3. Contract
4. Operations

Using Data Abstractions

(define p1 (make-point 1 2))
(define p2 (make-point 4 3))
(define s1 (make-seg p1 p2))

(define stretch-point
  (lambda (pt scale)
    (make-point
      (* scale (point-x pt))
      (* scale (point-y pt)))))

(stretch-point p1 2) => (2 . 4)
p1 => (1 . 2)
Abstractions Have Two Communities

- **Builders**
  
  ```scheme
  (define (make-point x y) (cons x y))
  (define (point-x point) (car point))
  ``

- **Users**
  
  ```scheme
  (* scale (point-x pt))
  ``

- Frequently the same person/people

- Schizophrenia can be a solid foundation for good programming style

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Elements of a Data Abstraction

-- Pair Abstraction --

1. **Constructor**

   ```scheme
   ; cons: A, B -> Pair<A,B> : A & B = anytype
   (cons <x> <y>) => <p>
   ``

2. **Accessors**

   ```scheme
   ; car: Pair<A,B> => A
   (car <p>) => <x>
   ``

   ```scheme
   ; cdr: Pair<A,B> => B
   (cdr <p>) => <y>
   ``

3. **Contract**

   ```scheme
   (car (cons <x> <y>)) => <x>
   (cdr (cons <x> <y>)) => <y>
   ``

4. **Operations**

   ```scheme
   ; pair?: anytype -> boolean
   (pair? <p>)
   ``

5. **Abstraction Barrier**

6. **Concrete Representation & Implementation**

   Could have alternative implementations!

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Rational Number Abstraction

- A rational number is a ratio n/d
  
  ```
  a/b + c/d = (ad + bc)/bd
  2/3 + 1/4 = (2*4 + 3*1)/12 = 11/12
  a/b * c/d = (ac)/(bd)
  2/3 * 1/3 = 2/9
  ```

---

-- Rational Number Abstraction --

1. **Constructor**

   ```scheme
   ; make-rat: integer, integer -> Rat
   (make-rat <n> <d>) -> <r>
   ``

2. **Accessors**

   ```scheme
   ; numer, denom: Rat -> integer
   (numer <r>)
   (denom <r>)
   ``

3. **Contract**

   ```scheme
   (numer (make-rat <n> <d>)) => <n>
   (denom (make-rat <n> <d>)) => <d>
   ``

4. **Operations**

   ```scheme
   ; print-rat r prints rat
   (print-rat <r>)
   ; +rat: Rat, Rat -> Rat
   (+rat x y)
   ; *rat: Rat, Rat -> Rat
   (*rat x y)
   ``

5. **Abstraction Barrier**

6. **Concrete Representation & Implementation**

   ```scheme
   ; Rat = Pair<integer,integer>
   (define (make-rat n d) (cons n d))
   (define (numer r) (car r))
   (define (denom r) (cdr r))
   ```
Rational Number Abstraction

1. Constructor
2. Accessors
3. Contract
4. Operations
5. Abstraction Barrier

6. Concrete Representation & Implementation
   ; Rat = List
   (define (make-rat n d) (list ___ ___))
   (define (numer r) (____ r))
   (define (denom r) (____ r))

Additional Rational Number Operations

; +rat: Rat, Rat -> Rat
(define +rat (rat x y)
  (make-rat (+ (* (numer x) (denom y)) (* (numer y) (denom x)))
            (* (denom x) (denom y))))

; *rat: Rat, Rat -> Rat
(define *rat (rat x y)
  (make-rat (* (numer x) (numer y))
            (* (denom x) (denom y))))

Using our system

• (define one-half (make-rat 1 2))
• (define three-fourths (make-rat 3 4))
• (define new (+rat one-half three-fourths))
  (numer new) ➝ 10
  (denom new) ➝ 8
  Oops - should be 5/4 not 10/8!!
"Rationalizing" Implementation

(define (gcd a b)
  (if (= b 0)
      a
      (gcd b (remainder a b))))

Strategy: remove common factors when access numer and denom

(define (numer r)
  (let ((g (gcd (car r) (cdr r))))
    (/ (car r) g)))

(define (denom r)
  (let ((g (gcd (car r) (cdr r))))
    (/ (cdr r) g)))

(define (make-rat n d)
  (cons n d))

Alternative "Rationalizing" Implementation

• Strategy: remove common factors when create a rational number

(define (numer r) (car r))

(define (denom r) (cdr r))

(define (make-rat n d)
  (let ((g (gcd n d)))
    (cons (/ n g) (/ d g))))

Either implementation is fine – most importantly no other code has to change if I switch from one to the other!!

Alternative +rat Operations

(define (+rat x y)
  (make-rat (+ (* (numer x) (denom y))
              (* (numer y) (denom x)))
              (* (denom x) (denom y))))

(define (+rat x y)
  (cons (+ (* (car x) (cdr y))
           (* (car y) (cdr x)))
       (* (cdr x) (cdr y))))

Abstraction Violation

Lessons learned

• Valuable to build strong abstractions
  • Hide details behind names of accessors and constructors
  • Rely on closure of underlying implementation
  • Enables user to change implementation without having to change procedures that use abstraction
  • Data abstractions tend to have procedures whose structure mimics their inherent structure

print-rat Operation

; print-rat: Rat -> undef
(define (print-rat rat)
  (display (numer rat))
  (display "/")
  (display (denom rat)))